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# Standing Waves in an Elastic Spring: A Systematic Study by Video Analysis

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The word "wave" is part of the daily language of every student. However, the physical understanding of the concept demands a high level of abstract thought. In physics, waves are oscillating variations of a physical quantity that involve the transfer of energy from one point to another, without displacement of matter. A wave can be formed by an elastic deformation, a variation of pressure, changes in the intensity of electric or magnetic fields, a propagation of a temperature variation, or other disturbances. Moreover, a wave can be categorized as pulsed or periodic.[1] Most importantly, conditions can be set such that waves interfere with one another, resulting in standing waves. These have many applications in technology, although they are not always readily identified and/or understood by all students. In this work, we use a simple setup including a low-cost constant spring, such as a Slinky, and the free software Tracker for video analysis. We show they can be very useful for the teaching of mechanical wave propagation and the analysis of harmonics in standing waves.

In teaching practice, we need to develop strategies and use tools to help students in the learning process. According to Moreira,[2] when new information has little to no interaction with previous concepts, very little to nothing is introduced in the cognitive structure and significant learning does not occur.

Thus, the use of technology and resources in experimentation goes beyond a simple contextualization. It can create operational situations as a basis for understanding abstract concepts.[2-4] Video modeling is used to facilitate the understanding of physics,[5] primarily in the study of kinematics,[6,7] and in other areas such as sound waves[8] and optics.[9]

## Experimental

An elastic spring is a medium where vibrations can easily be produced. Under identical end conditions, e.g., both ends tied down, the spring can vibrate with standing waves with a variety of wavelengths, known as harmonics.

To study the mechanical waves, a black painted metal spring was fixed to a support. The spring was illuminated by white light projectors to enable a good contrast of the spring against a white background, as suggested by Sirisathitkull et al.[6] To capture the video images, we used a digital photo camera Canon EOS 5D Mark III with HD 1280×720 MP resolution at 60 frames per second and a CMOS Full Frame 36× 24 mm$^2$ sensor. A smartphone or tablet camera can also be used, but images are frequently blurred for the higher order harmonics due to their small CMOS sensor dimensions. In order to quantitatively characterize the waves, a calibrated reference bar was included in the videos.

## Standing waves in an elastic spring

To excite the harmonics in the elastic spring, one end of the spring is tied down while the other is propelled manually, producing periodic pulses perpendicular to the elastic spring alignment. Although the experiment does not recreate exactly the desired boundary condition that requires both ends to be fixed for generating standing waves, the superposition of the pulses that travel along the spring in opposite directions yields standing waves only at specific frequencies. The constructive interferences generated antinodes, whereas the destructive interference generated nodes. Students have fun when generating the standing waves with their own hands, and they easily identify the waves' stationary behavior because visually there is no change in the position of nodes and antinodes with time.

A video with sequences of stationary waves was recorded at 60 frames per second and is available online.[10] Figure 1 is a snapshot of the experiment when the third harmonic was being formed.

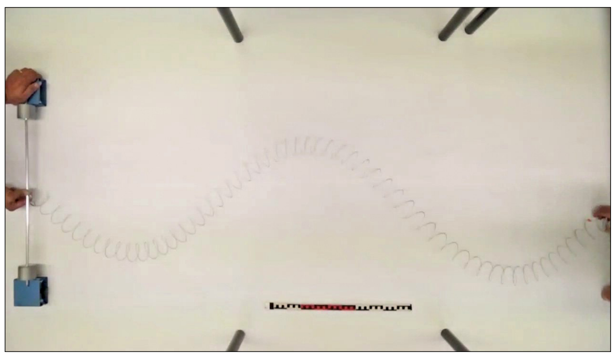

**Fig. 1. Image of the spring vibration when the third harmonic was being formed. The left side of the spring is fixed while the right side is propelled manually. The calibration bar is visible at the bottom of the image.**

By observing the sequence frame-by-frame with the Tracker[11] software, students realize that the distance between two consecutive nodes corresponds to one-half of the wavelength of the stationary wave. This conclusion is not evident when students only observe schemes in textbooks of standing waves, similar to those in Fig. 2. With video analysis, students can actually measure the approximate wavelength of each standing wave with the "tape measure" tool from the software. The images in Fig. 2 are strobe pictures of the harmonics recorded. They can be built with the "ghost" filter from Tracker. Teachers can use the images in Fig. 2 to discuss conceptually with the students how standing waves are formed and why they look like they do. Moreover, we show

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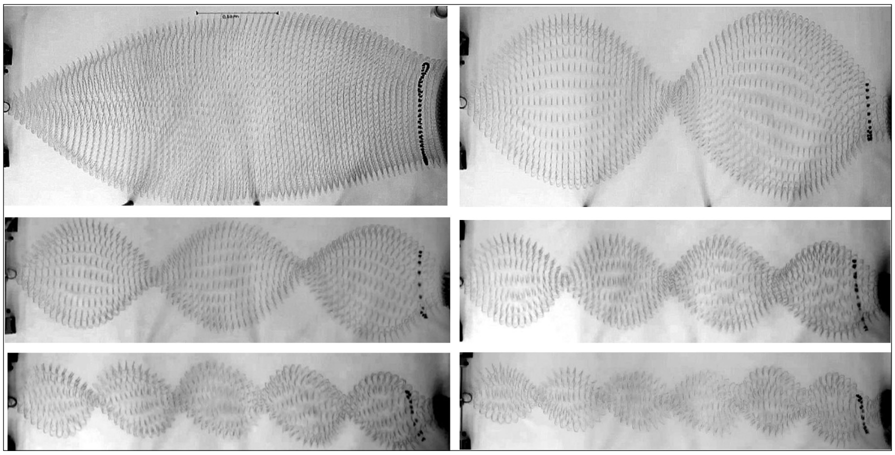

**Fig. 2. Stroboscope images of the spring vibration, from the first (top left-hand) to the sixth (bottom right-hand) harmonic. The wavelength is measured by using the calibration bar within the videos, which is not visible in all of the zoomed-in images shown.**

in the following how video analysis enables students to determine the period and the speed of the propagation of waves in the spring.

## Quantitative analysis of the harmonics

The vibration modes typical for an elastic string depend on its tension and length. Equation (1) represents the frequency of the harmonics when both ends are tied down:

$$f_n = \frac{nV}{2L}, \tag{1}$$

where $n$ is the harmonic order, $V$ is the propagation speed of the wave in the elastic medium, and $L$ is the total length of the string,[1] assuming that its tension is the same during the whole experiment.

For simplicity, it may be assumed that, on average, the tension of the spring between the two ends is nearly constant, and thus the frequencies of the harmonics generated in the spring can be approximately described by Eq. (1).

With the software package, students can obtain the wavelength ($\lambda$) measuring the distances between nodes (in most cases, an extrapolation from the shape of the images in Fig. 1 is needed to determine the position where the right end node would be if the wave is extended), and the period ($T$) for each harmonic (the period is measured for a complete oscillation of the harmonic wave). They can relate them by the well-known expression from kinematics:

$$\lambda_n = V\,T_n = \frac{V}{f_n}. \tag{2}$$

The plot of the wavelength as a function of the wave period enables the determination of the propagation speed of the wave in the spring. This is an important step for students' understanding of wave propagation in standing waves, because these are usually presented to the students as "stationary" waves. The experimental values of $V$ obtained from the experiment can vary from group to group, according to the tension and/or the type of spring used in the experiment; we thus strongly suggest to teachers that they provide different springs (with different linear density) in order to explore this situation.

Figure 3 presents an example of a plot of $\lambda$ against $T$ for the first six harmonics of Fig. 2. The values of $\lambda$ are affected by an estimated error of 10%, due to the extrapolation that is needed for the position of end node at the right side of the spring. A linear relationship can be observed, and it is expected that students conclude that such a linear behavior confirms the physical model assumed, and the results predicted by Eq. (2).

Teachers can also suggest that their students compare the speed obtained from the slope in Fig. 3 with the direct measurement of the speed of a traveling wave pulse, which would also be easy to obtain using Tracker. The collected data are listed in Table I.

The speed of the propagation of the waves in the spring is calculated by a linear fit to the experimental data, either with Tracker or another software with statistics tools (for example, MS Excel). The obtained value in our example is $v = 4.95 \pm 0.02$ m/s. As discussed above, this value depends greatly on

**Table I. Experimental values of the period and the wavelength measured for each harmonic.**

| Harmonic $n$ | Period $T$ (s) ($\pm$ 0.02 s) | Wavelenth $\lambda$ (m) ($\pm$10%) |
|---|---|---|
| 1st | 1.12 | 5.5 |
| 2nd | 0.52 | 2.6 |
| 3rd | 0.35 | 1.7 |
| 4th | 0.27 | 1.3 |
| 5th | 0.21 | 1.0 |
| 6th | 0.18 | 0.89 |

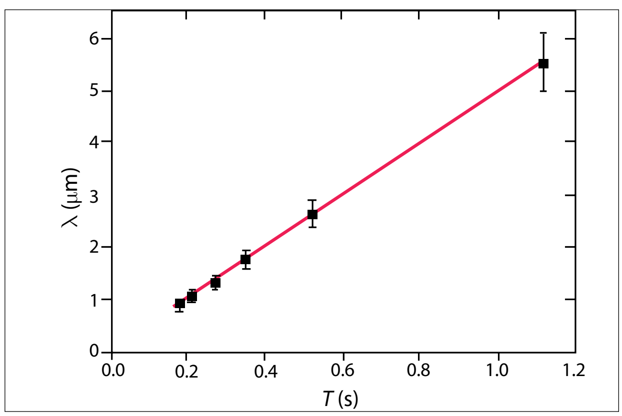


**Fig. 3. The wavelength as a function of the period for the first six harmonics of Fig. 2. The slope of the linear fit gives the speed of the propagation of waves in the spring. The error bars correspond to an estimated error of about 10% in the measurement of $\lambda$.**

the elastic characteristics and/or the tension of the spring. If these parameters are intentionally varied, it can lead to additional interesting experimentation and improve students' understanding.

## Conclusions

What is important to highlight in this work is the possibility to generate transverse standing mechanical waves propelled manually, using easily accessible and low-cost material like elastic springs. Moreover, Tracker is a freeware software that allows the collection of experimental data by video analysis and data modeling with the built-in "Data Tools" module; no additional sophisticated digital and technological resources are needed to record the standing waves, although good quality digital recording for quantitative measures may require a $200 to $300 camera.

The experiment is easy to execute in any classroom and, as much as possible, it should be implemented as an inquiry-based activity rather than a cookbook-style activity. By doing this, teachers provide a hands-on approach to take advantage of students' engagement in the activity, and involve them in the discussion of the results. We suggest this approach should be at first qualitative, and then followed by a quantitative interpretation of the phenomenon.

Producing stationary waves makes possible a quick identification of the constructive (antinodes) and destructive (nodes) interference points. The strobe images of the oscillation of the spring allow the observation of the motion of the medium in the antinode regions, which are frequently not perceived by the students when they see a typical image of a stationary wave in textbooks.

A video analysis and a computer simulation are sometimes assumed as virtual-based experiments, because both share digital tools and show comparable characteristics (can be run at any time and as many times as needed, allow data acquisition with virtual tools, can be explored inside or outside the classroom individually or in groups). Nevertheless, they have different impacts on students' engagement: students tend to "believe" more in videos because they are real experiments recorded digitally, whereas simulations are animations based on a computational algorithm. Moreover, they know the origin of experimental results is quite different. Students can even make their own videos as hands-on activities, and thus they realize that with a video the results are obtained from real experimental conditions and are not generated by a mathematical algorithm.

The video of this experience, available on online,[10] may also be explored as an activity of curricular enrichment outside the classroom, and then later be discussed and complemented at the school's laboratory. In addition, for those teachers without resources or access to laboratory infrastructures, this video enables them to explore the topic of stationary waves in an affordable way.


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